Tanta University

Department: Engineering Physics and Mathematics
Total Marks: 85 Marks

Faculty of Engineering

Course Title: Mathematics(3B)	Course Code: PME2211	Year: 2nd(Computer &Control Dep.)
Date: June 8th, 2014 (Second term)	Allowed time: 3 Hrs	No. of Pages: (3)

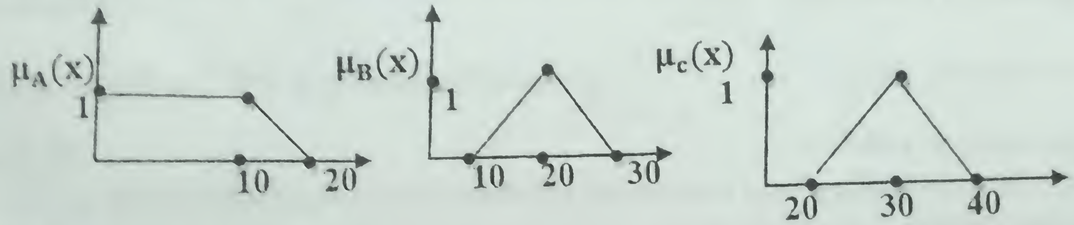
Remarks: Answer All of The Following Questions

Question Number (1) (25 Marks)

(a) Show that the set $A = \int \left(\frac{1}{\sqrt{1+5x}}\right)/_x$ is convex

(b) Consider the fuzzy sets F and G defined in interval [0,10] by the memberships $\mu_F(x) = 2^{-x}$ and $\mu_G(x) = \frac{1}{1+10(x-2)^2}$ .Determine the mathematical formulas and graphs of memberships functions of (i) $\mu_{\bar{F}}$ and $\mu_{\bar{G}}$ (ii) $\mu_{F\cup G}$ and $\mu_{F\cap G}$

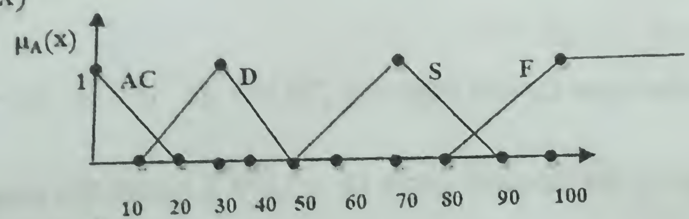
(c) A product with memberships represents, degree of high expensive $\mu_A(x)$, degree of medium expensive $\mu_B(x)$ and degree of cheap expensive $\mu_C(x)$ .Us defuzzification methods to find suitable price, if its medium degree is 0.5 and high degree 0.8 where

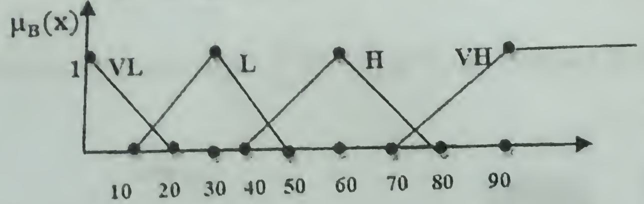


(d) Consider Washing machine with two input and one output . The input :

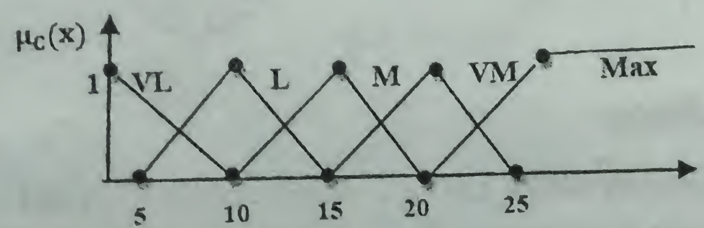
[1] The dirtiness of the Load which measured by the opacity of the washing water use an optical sensor system {Almost clean(AC), Dirty(D), Soiled(S), Filthy(F)}with fuzzy dirtiness membership $\mu_A(x)$

[2] The weight of the Laundry load as measured by a pressure sensor system {Very light(VL), Light(L), Heavy(H), Very heavy(VH)} with fuzzy weight membership $\mu_B(x)$





The output is the amount of detergent dispensed {Very little(VL), Liittle(L), Much(M), Very much(VM), Maximum(Max)} $\mu_C(x)$



Find the fuzzy detergent dispensed value if laundary has dirtiness values 13 and weight 72

Question Number (2) **(15 Marks)**

If the method of Frobenius is used to solve the following linear homogenous 2nd order ordinary differential equation (L.H.O.D.E.)

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+(x^2+1)y=0$$

1. Show that the point x=0 is the unique regular singular point. (3 Marks)
2. Deduce the indicial equation and find the values of λ. (4 Marks)
3. Deduce the recurrence relation for coefficient of the series solution. (4 Marks)
4. Deduce the general series solution y(x). (4 Marks)

Question Number (3) **(15 Marks)**

1. Sketch the domain of the function $f(z)=\frac{1}{3(z^2+\bar{z}^2)-10z\bar{z}+25}$. (3 Marks)
2. Prove that $f(z)=\frac{z}{z^4+1}$ is continuous at all points inside and on the unit circle $|z|=1$ except at some points and determine these points. (3 Marks)
3. Discuss the existence of the limit $\lim\limits_{z\to0}\frac{\bar{z}^2}{z}$. (3 Marks)
4. Show that the function $f(z)=\begin{cases}\frac{\bar{z}^2}{z} & z\neq0\\ 0 & z=0\end{cases}$ is not differentiable at z=0 but the Cauchy-Riemann equations are satisfied. (3 Marks)
5. Construct the Riemann surface and the branch structure for the function $f(z)=\sqrt{z}\,\sqrt[3]{z}$ if the angle is embedded in $[-\pi,\pi]$. (3 Marks)

Question Number (4) **(15 Marks)**

1. Let f(z) be an analytic in a simple connected domain and let C be a simple closed contour lying entirely within the domain, if z_o is any point interior in C then prove that $2\pi i\, f^{(n)}(z_o)=n!\oint_C \frac{f(z)}{(z-z_o)^{n+1}}dz,\ n=0,1,2,3,\dots$. (3 Marks)
2. Evaluate $\oint_C \frac{Ln(z)}{(z-3)^3(z+4)^2}dz,\ C:|z|=3.$ (3 Marks)
3. Verify Cauchy's integral theorem for the function $f(z)=2i$ if C is $|z-3i|+|z+3i|=20$. (3 Marks)
4. Find and sketch the image of the circular curve $|z-1|=1$ under the mapping $w=\frac{1}{z}$. (3 Marks)
5. Find the bilinear mapping that maps $0\le Arg\{z\}\le\frac{\pi}{4}$ onto the unit circle $|w|\le1$ as shown in the following figure (4.5). (3 Marks)

Figure (4.5)

Question Number (5) **(15 Marks)**

1. Let f(z) be an analytic function within and on a closed contour C except at a finite number of singular points $z_1, z_2, \dots, z_n$ interior to C. Then prove $\oint f(z)dz=2\pi i\sum_{i=1}^{n}\underset{z=z_i}{Res}\{f(z)\}$ where the integral is taken counter clockwise direction around C. (3 Marks)

2. Find the Laurent series for $f(z) = \frac{z-e^{2z}}{z-1}$ about z=1 then name the singularity and give the region of the convergence. (3 Marks)

3. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $0 < |z+1| < 2$ and find its residue. (3 Marks)

4. Using the residue theorem to evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$. (3 Marks)

5. Using the residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} dx$. (3 Marks)

With Best Wishes

Course Examination Committee and Course Coordinators

Dr. Mohamed Shokry and Dr. Mohamed Elborhamy and the committee